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Driven asymmetric passages in a two-well system

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Abstract : Interesting results are obtained when a particle(subsystem) in a double-well potential is subjected to an external asymmetric periodic driving force. The asymmetry is with respect to the rate of change of the field in time in a period. In our model the numerical calculations show that the hysteresis loss of the system is very sensitive to the degree of asymmetry of the field. The hysteresis loss, in turn, is defined to be a measure of synchronized passages of the system from one well of the potential to the other. We remark, in conclusion, that the asymmetry of the driving field may provide a clue to the preferentially unidirectional motion of a Brownian particle (a biological macromolecule, for example) in a periodic potential system (say, along a microtubule) when subjected to such an asymmetric but zero time averaged external periodic field.

Key words: asymmetric motion, hysteresis, first-passage-time.

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A net (statistically) asymmetric motion (of macromolecules) has recently been observed experimentally in biological systems (along microtubules)[1]. Some simple physical models have been put forth to understand (although none claim to mimic) the observed behavior. These models[2] often assume particle to move in a spatially asymmetric but periodic potential subjected to colored noise or externally applied temporally periodic field. These are interesting models from the physics point of view and often find application as a bonus in various branches of science[3]. In the present work we propose that an asymmetric motion can be realized in a spatially symmetric periodic potential system subjected to white noise fluctuations when a temporally asymmetric but zero time-averaged periodic external field is applied to it. The proposal draws an obvious inspiration from the motion of sperm molecules of eucaryotes. The two distinct (power and reverse) strokes of the tails of the sperm molecules help the molecule move backward and forward in a viscous fluid. These two strokes have predominantly two distinct time scales of flapping. We set to prove our proposition numerically with the help of a hysteretic system where the hysteresis loss is governed by how synchronous the response of the system parameters are with respect to the externally applied temporally periodic field. We take this externally periodic field to be temporally asymmetric and, to be specific, we take a saw-tooth type field with two different ±slopes (time scales).

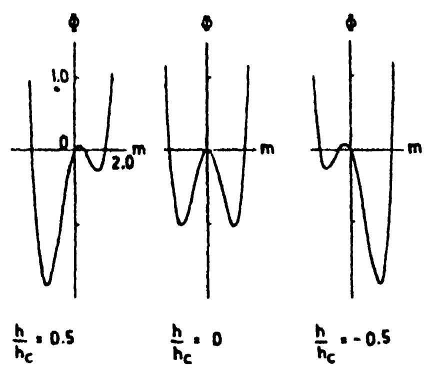

Figure 1. Shows $\Phi(m) = U(m) + mh$ for various h values.

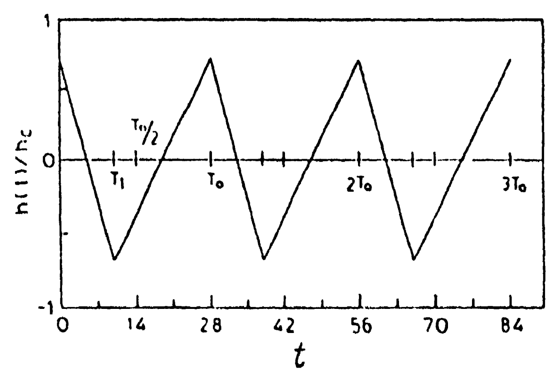

Figure 2. Depicts the external periodic field $h(t)$. We have taken amplitude $h_0 = 0.7\,h_c$.

We take a two well (Landau) potential [Fig.1]

$$U(m) = -m^2 + \frac{1}{4}m^4, \tag{1}$$

to represent our working(substrate) system on which a subsystem(particle) moves as response to the external saw-tooth type field [Fig.2]. An ensemble of such non-interacting subsystems subjected to white noise fluctuations provides us information about the average behavior. With a little generalization, our results can be extrapolated to be valid for a periodic potential system. We numerically calculate the first passage time distribution $\rho(\tau)$ of passages of the particle from one well to other by solving overdamped Langevin equation

$$\dot{m} = -\frac{\delta\phi(m)}{\delta m} + f(t), \tag{2}$$

where the noise $f(t)$ obeys the statistics

$$< f(t) >= 0, \tag{3a}$$

and

$$< f(t)f(t') >= 2D\delta(t-t'). \tag{3b}$$

The distribution $\rho(\tau)$ (a typical one being shown in Fig.3) helps us calculate half of the hysteresis loop

$$M(h_J) = 1 - \frac{2}{h_c}\int_{h_J}^{h_0} \rho(h_J')dh_J', \tag{4}$$

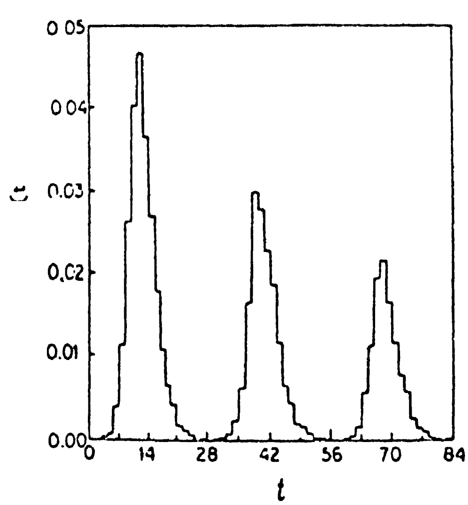

Figure 3. Part of the first-passage time distribution $\rho(\tau)$ for the field sweep shown in Fig.2. $\rho(\tau)$ extends to 24 cycles of $h(t)$ in 15000 runs for $D = 0.5$ and $h_0 = 0.7\ h_c$.

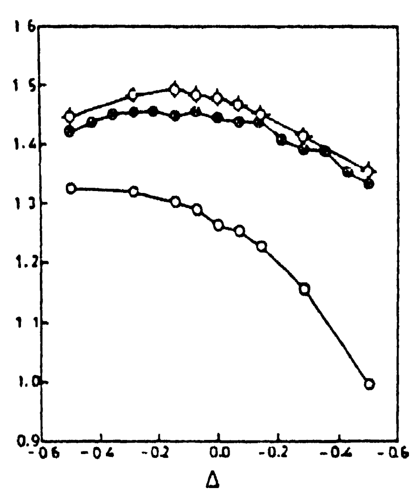

Figure 4. Hysteresis loop area *versus* the asymmetry $\Delta = \frac{(T_1 - T_0/2)}{T_0/2}$ in the $\pm$ slopes of $h(t)$ (Fig.2.) for $D = 0.3$ (●), $D = 0.5$ (⊗) and $D = 0.7$. For each data point $\rho(\tau)$ calculated from a minimum of 10000 first-passage-times τ from the right well to the left well were used.

with the saturation value of $\frac{M}{h_c} = \pm 1$ and where $\rho(h_J)$ is the distribution of jump field values h_J obtained from $\rho(\tau)$. h_c is the critical field value at which one of the two wells of (1) disappears and $h_0(< h_c)$ is the amplitude of the saw-tooth field sweep. The other half of the hysteresis loop is obtained by symmetry. The area of the hysteresis loop so obtained shows asymmetric behavior [Fig.4] with respect to the asymmetry parameter $\Delta = \frac{(T_1 - T_0/2)}{T_0/2}$. The symbols T_1 and T_0 are explained in Fig.2. From Fig.4 it is obvious that hysteresis loss or synchronized behavior of passage differs depending on how asymmetrically the field is swept. When the field is swept with asymmetry Δ the passage becomes more synchronized in one direction than the other whereas for asymmetry $-\Delta$ the behavior is qualitatively the reverse. It should be noted that the time average of the field sweep in a period is always zero even for the asymmetric cyclic field yet the passage in one direction is preferred to the passage in the other direction. This asymmetric behavior of passages may provide some hint to understand the observed behavior of biological system, referred to in the beginning of this paper, at the most preliminary level.

The hysteresis loss calculated in such a two well system subjected to a symmetric saw-tooth type periodic drive field shows the phenomenon of stochastic resonance with respect to the white noise strength[4]. It would be quite interesting to examine whether the same phenomena can be observed when the system is driven by asymmetric saw-tooth type periodic field. Some work in this direction is currently underway.

References

[1] K. Svoboda, C.F. Schmidt, B.J. Schnapp, and S.M. Block, *Nature* **365**, 721 (1993).

[2] See, for example, C.R. Doering, W. Horsthemke, and J. Riordan, *Phys. Rev. Lett.* **72**, 2984 (1994) and references therein.

[3] J. Rousselet, L. Salome, A. Ajdari, and J. Prost, *Nature* **370**, 446 (1994).

[4] M.C. Mahato and S.R. Shenoy, *Phys. Rev.* **E50**, 2503 (1994).